Efficient N-particle W state concentration with different parity check gates

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We present an universal way to concentrate an arbitrary N-particle less-entangled W state into a maximally entangled W state with different parity check gates. It comprises two protocols. The first protocol is based on the linear optical elements say the partial parity check gate and the second one uses the quantum nondemolition (QND) to construct the complete parity check gate. Both of which can achieve the concentration task. These protocols have several advantages. First, it can obtain a maximally entangled W state only with the help of some single photons, which greatly reduces the number of entanglement resources. Second, in the first protocol, only linear optical elements are required which is feasible with current techniques. Third, in the second protocol, it can be repeated to perform the concentration step and get a higher success probability. All these advantages make it be useful in current quantum communication and computation applications.

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I. INTRODUCTION

Entanglement is the important quantum resource in both quantum communication and computation [1, 2]. The applications of entanglement information processings such as quantum teleportation [3, 4], quantum key distribution (QKD) [5–7], quantum dense coding [8, 9], quantum secret sharing [10–12] and quantum secure direct communication (QSDC) [13–15] all resort the entanglement for setting up the quantum channel between long distance locations. Unfortunately, during the practical transmission, an entangled quantum system can not avoid the channel noise that comes from the environment, which will degrade the entanglement. It will make a maximally entangled state system become a mixed one or a partially entangled one. Therefore, these nonmaximally entangled systems will decrease the security of a QKD protocol if it is used to set up the quantum channel. Moreover, they also will decrease the fidelity of quantum dense coding and quantum teleportation.

Entanglement purification is a powerful tool for parties to improve the fidelity of the entangled state from a mixed entangled ensembles [16–28]. On the other hand, the entanglement concentration protocol (ECP) is focused on the pure less-entangled system, which can be used to recover a pure less-entangled state into a pure maximally entangled state with only local operation and classical communications [29–43]. Most of the ECPs such as the Schmidt decomposition protocol proposed by Bennett *et al.* [29], the ECPs based on entanglement swapping [30, 31], linear optics [32–34], and cross-Kerr nonlinearity [35, 36] are all focused on the Bell states and multi-partite Greenberger-Horne-Zeilinger (GHZ) states. Because all the ECPs for Bell stats can be easily extended to the GHZ states.

On the other hand, the W state, which has the different entanglement structure and can not be convert to the GHZ state directly with only local operation and classical communication, has began to receive attention both in theory and experiment [44–49]. Agrawal and Pati presented a perfect teleportation and superdense coding with W states in 2006 [45]. In 2010, Tamaryan *et al.* discussed the universal behavior of the geometric entanglement measure of many-qubit W states [47]. Eibl *et al.* also realized a three-qubit entangled W state in experiment [48]. Several ECPs for less-entangled W state were also proposed [39–43]. In 2003, Cao and Yang has discussed the W state concentration with the help of joint unitary transformation[39]. In 2007, a W state ECP based on the Bell-state measurement has been proposed [40]. Then in 2010, Wang *et al.* have proposed an ECP which focuses on a special kind of W state [41]. Recently, Yildiz proposed an optimal distillation of three-qubit asymmetric W states [42]. We also have proposed

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an ECP with both linear optics and cross-Kerr for three-particle W state [43]. Unfortunately, these ECPs described above all focus on the three-particle W state and they are mostly to concentrate some W states with the special structures.

In this paper, we will present an ECP for arbitrary multi-partite polarized W entangled systems. We will describe this protocol in two different ways. First, we use the partial parity check (PPC) gate constructed by linear optics to perform this protocol. Second, we introduce the complete parity check (CPC) gate to achieve this task. Compared with other conventional ECPs, we only resort the single photon as an auxiliary which largely reduce the consumed quantum resources. Moreover, with the help of CPC gate, this protocol can be repeated and get a higher success probability. This paper is organized as follows: in Sec. II, we first briefly explain our PPC gate and CPC gate. in Sec. III, we describe our ECP with both PPC and CPC gates respectively. In Sec. IV, we make a discussion and summary.

II. PARITY CHECK GATE

Before we start to explain this protocol, we first briefly describe the parity check gate. Parity check gate is the basic element in quantum communication and computation. It can be used to construct the controlled-not (CNOT) gate [50, 51]. It also can be used to perform the entanglement purification [20, 22] and concentration protocol [32, 33].

A. partial parity check gate

There are two different kinds of parity check gates. One is the partial parity check (PPC) gate and the other is the complete parity check (CPC) gate . In optical system, a polarization beam splitter (PBS) is essentially a good candidate for PPC gate as shown in Fig.1. Suppose that two polarized photons of the form

$$|\varphi_1\rangle = \alpha|H\rangle + \beta|V\rangle, |\varphi_2\rangle = \gamma|H\rangle + \delta|V\rangle, \tag{1}$$

entrance into the PBS from different spatial modes. Here $|\alpha|^2+|\beta|^2=1$, and $|\gamma|^2+|\delta|^2=1$. $|H\rangle$ and $|V\rangle$ represent the horizonal and the vertical polarization of the photons, respectively.

Let $|\varphi_1\rangle$ be in the spatial mode a_1 and $|\varphi_2\rangle$ be in the spatial mode a_2. The whole system can be described as

$$\begin{aligned} |\varphi_1\rangle &\otimes |\varphi_2\rangle = (\alpha|H\rangle_{a_1} + \beta|V\rangle_{a_1}) \otimes (\gamma|H\rangle_{a_2} + \delta|V\rangle_{a_2}) \\ &= \alpha\gamma|H\rangle_{a_1}|H\rangle_{a_2} + \beta\delta|V\rangle_{a_1}|V\rangle_{a_2} \\ &+ \alpha\delta|H\rangle_{a_1}|V\rangle_{a_2} + \beta\gamma|V\rangle_{a_1}|H\rangle_{a_2} \end{aligned} \tag{2}$$

Then after passing through the PBS, it evolves as

$$\begin{aligned} &\rightarrow \alpha\gamma|H\rangle_{b_1}|H\rangle_{b_1} + \beta\delta|V\rangle_{b_1}|V\rangle_{b_2} \\ &+ \alpha\delta|H\rangle_{b_1}|V\rangle_{b_1} + \beta\gamma|V\rangle_{b_2}|H\rangle_{b_2}. \end{aligned} \tag{3}$$

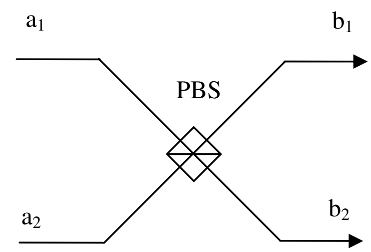

FIG. 1: A schematic drawing of our PPC gate. It is constructed by a polarization beam splitter(PBS). It is used to transfer a $|H\rangle$ polarization photon and to reflect a $|V\rangle$ polarization photon.

From above description, items $|H\rangle_{b_1}|H\rangle_{b_1}$ and $|V\rangle_{b_1}|V\rangle_{b_2}$, say the even parity states will lead the output modes b_1 and b_2 both exactly contain only one photon. But items $|H\rangle_{b_1}|V\rangle_{b_1}$ and $|V\rangle_{b_2}|H\rangle_{b_2}$ will lead the two photons be in the same output mode, which cannot be distinguished. Based on the post selection principle, only the even parity state is the successful case. Therefore, the total success probability is $|\alpha\gamma|^2 + |\beta\delta|^2 < 1$. This is the reason that we call it PPC gate.

B. complete parity check gate

Another parity check gate say CPC gate is shown in Fig. 2. We adopt the cross-Kerr nonlinearity to construct the CPC gate. Cross-Kerr nonlinearity has been widely used in quantum information processing[53–56]. In general, the Hamiltonian of the cross-Kerr nonlinearity is described as $H = \hbar\chi\hat{n_a}\hat{n_b}$, where the $\hbar\chi$ is the coupling strength of the nonlinearity. It is decided by the material of cross-Kerr. The $\hat{n_a}(\hat{n_b})$ are the number operator for mode $a(b)$[51, 52].

Now we reconsider the two photon system $|\varphi_1\rangle \otimes |\varphi_2\rangle$ coupled with the coherent state $|\alpha\rangle$. From Fig. 2, the whole system evolves as

$$\begin{aligned} |\varphi_1\rangle &\otimes| \varphi_2\rangle \otimes |\alpha\rangle = (\alpha\gamma|H\rangle_{a_1}|H\rangle_{a_2} + \beta\delta|V\rangle_{a_1}|V\rangle_{a_2} \\ &+ \alpha\delta|H\rangle_{a_1}|V\rangle_{a_2} + \beta\gamma|V\rangle_{a_1}|H\rangle_{a_2}) \otimes |\alpha\rangle \\ &\rightarrow (\alpha\gamma|H\rangle_{b_1}|H\rangle_{b_1} + \beta\delta|V\rangle_{b_1}|V\rangle_{b_2})|\alpha\rangle \\ &+ \alpha\delta|H\rangle_{b_1}|V\rangle_{b_1}|\alpha e^{-i2\theta}\rangle + \beta\gamma|V\rangle_{b_2}|H\rangle_{b_2}|\alpha e^{i2\theta}\rangle. \end{aligned} \tag{4}$$

It is obvious to see that the even parity states make the coherent state $|\alpha\rangle$ pick up no phase shift, but the odd parity state $|H\rangle_{b_1}|V\rangle_{b_2}$ makes the coherent state pick up the phase shift -2θ. The other odd parity state $|V\rangle_{b_1}|H\rangle_{b_2}$ make the coherent state pick up the phase shift with 2θ. With a general homodyne-heterodyne measurement, the phase shift 2θ and -2θ can not be distinguished[51].

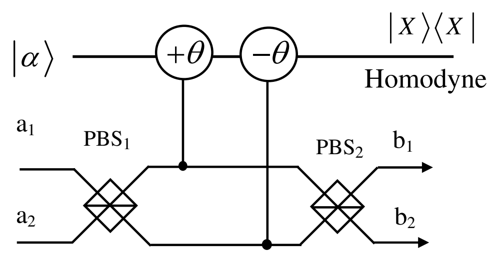

FIG. 2: A schematic drawing of our CPC gate.

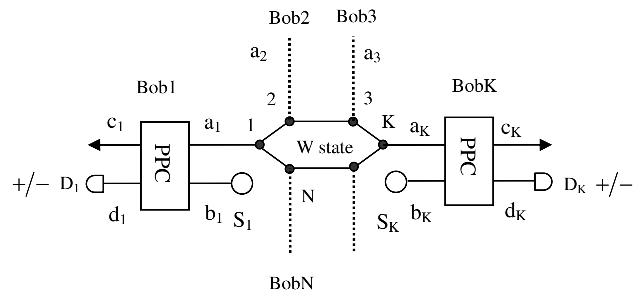

FIG. 3: A schematic drawing of our ECP with PPC gate. Each parties except Bob2 own the PPC gate and perform the parity check. If they pick up the even parity state, it is successful, otherwise, it is a failure.

Then one can distinguish the different parity state according to their different phase shifts. So the success probability of the initial states collapsing to the even and odd parity state is $|\alpha\gamma|^2+|\beta\delta|^2+|\alpha\delta|^2+|\beta\gamma|^2=1$, in principle. So we call it CPC gate. Compared with the PPC gate, the success probability for CPC gate can reach the max value 1 but the PPC gate cannot reach 1. Another advantage of the CPC gate is that we get the both even and odd parity state by measuring the phase shift of the coherent state. That is to say, we do not need to measure the two photons directly. So after the measurement, the two photons can be remained. It is so called quantum nondemolition(QND) measurement. But in PPC gate, we should use the post selection principle to detect the two photons being in the different spatial modes by coincidence counting. After both detectors register the photons with a success case, the photons are destroyed and cannot be used further more.

III. N-PARTICLE LESS-ENTANGLED W STATE CONCENTRATION WITH PARITY CHECK GATE

A. N-particle less-entangled W state concentration with PPC gate

In this section, we start to describe our N-particle ECP with PPC gate. An N-particle W state can be described as

$$\begin{aligned}|\Psi\rangle_N &= \alpha_1|V\rangle_1|H\rangle_2|H\rangle_3+\cdots+|H\rangle_{N-1}|H\rangle_N\\ &+ \alpha_2|H\rangle_1|V\rangle_2|H\rangle_3+\cdots+|H\rangle_{N-1}|H\rangle_N\\ &+ \cdots+\alpha_N|H\rangle_1|H\rangle_2+\cdots+|H\rangle_{N-1}|V\rangle_N\\ &= \alpha_1|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}+\alpha_2|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\ &+ \cdots+\alpha_N|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N. \qquad (5)\end{aligned}$$

where $|\alpha_1|^2+|\alpha_2|^2+\cdots+|\alpha_N|^2=1$. In order to explain this ECP clearly simply, we let α_1, α_2, $\cdots$ be real. Certainly, this ECP is also suitable for the case of α_1, α_2, $\cdots$ being complex. $|\widetilde{H}\rangle^{N-2}$ means that the $N-2$ photons say $|H\rangle_3|H\rangle_4\cdots|H\rangle_N$ are all in the $|H\rangle$ polarization.

From Fig. 3, the N-photon less-entangled W state of the form Eq.(5) is distributed to N parties, saies Bob1, Bob2, $\cdots$, Bob$N$. Bob1 receives the photon of number 1 in the spatial mode a_1. Bob2 receives the number 2 in the spatial mode a_2, and Bob$N$ receives the photon number N in the spatial mode a_N. That is to say, each of the parties owns one photon.

The principle of our ECP with PPC gate is shown in Fig. 3. The basic idea of realizing the concentration is to use the local operation and classical communication to make each coefficients on each items of Eq. (5) all equal to α_2. If all coefficients are equal, they can be regarded as a common factor and can be neglected. The remained state is essentially the maximally entangled W state. Thus, the whole process can be briefly described as follows: we first divide the whole procedure into $N-1$ steps. In each step, each party say Bob$K$ should first prepare a single photon. In Fig. 3, the single-photon sources S_1, S_3, $\cdots$, S_K, $\cdots$, S_N are used to prepare the single photons locally. Then he performs a parity check measurement for his two photons. The one comes from the single photon he prepared, and the other is the photon from the less-entangled W state. If the parity check measurement is successful, then he asks the other to perform the further operation.

Bob1 first perform the parity check on the photon of number 1 and the prepared single photon. The single-photon resource S_1 for Bob1 prepares a single photon in the spatial mode b_1 of the form

$$|\Phi\rangle_1=\frac{\alpha_1}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle+\frac{\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle. \qquad (6)$$

Then the initial less-entangled W state $|\Psi\rangle_N$ combined with $|\Phi\rangle_1$ can be described as

$$\begin{aligned}&|\Psi\rangle_{N+1}=|\Psi\rangle_N\otimes|\Phi\rangle_1=(\alpha_1|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\ &+\ \alpha_2|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}+\cdots+\alpha_N|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N)\\ &\otimes\ (\frac{\alpha_1}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle+\frac{\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle)\\ &=\ \frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\end{aligned}$$

$$
\begin{aligned}
&+ \frac{\alpha_2^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_1\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_1\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\\
&+ \cdots\\
&+ \frac{\alpha_1\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N\\
&+ \frac{\alpha_1\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\\
&+ \cdots\\
&+ \frac{\alpha_2\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N. \qquad (7)
\end{aligned}
$$

After passing through the PPC gate in Bob1's location, Bob1 only picks up the even parity state in the spatial mode c_1 and d_1. Therefore, the above state collapses to

$$
\begin{aligned}
|\Psi\rangle'_{N+1} &= \frac{\alpha_1\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_1\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\\
&+ \cdots\\
&+ \frac{\alpha_1\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N\\
&+ \frac{\alpha_1\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}. \qquad (8)
\end{aligned}
$$

It can be rewritten as

$$
\begin{aligned}
|\Psi\rangle''_{N+1} &= \frac{\alpha_2}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|V\rangle|V\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_3}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|H\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\\
&+ \cdots\\
&+ \frac{\alpha_N}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|H\rangle|H\rangle_1|\widetilde{H}\rangle^{N-2}|V\rangle_N. \qquad (9)
\end{aligned}
$$

Finally, Bob1 measures the photon in the spatial mode d_1 (the first photon in Eq. (9)) in the basis $|\pm\rangle$, with $|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$. Then they will get

$$
\begin{aligned}
|\Psi^{\pm}\rangle_N^1 &= \pm\frac{\alpha_2}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_3}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\\
&+ \cdots\\
&+ \frac{\alpha_N}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}}|H\rangle_1|\widetilde{H}\rangle^{N-2}|V\rangle_N. \qquad (10)
\end{aligned}
$$

The superscription 1 means that they perform the concentration on the first particle. If the measurement result is $|+\rangle$, they will get $|\Psi^+\rangle_N^1$. If the result is $|-\rangle$, they will get $|\Psi^-\rangle_N^1$. In order to get $|\Psi^+\rangle_N^1$, one of the parties, Bob1, Bob2, $\cdots$ should perform a local operation of phase rotation on his photon. The total success probability is

$$
\begin{aligned}
P^1 &= \frac{2\alpha_1^2\alpha_2^2+\alpha_1^2(\alpha_3^2+\alpha_4^2+\cdots+\alpha_N^2)}{\alpha_1^2+\alpha_2^2}\\
&= \frac{\alpha_1^2(2\alpha_2^2+\alpha_3^2+\alpha_4^2+\cdots+\alpha_N^2)}{\alpha_1^2+\alpha_2^2}. \qquad (11)
\end{aligned}
$$

Compared with Eq.(5), the coefficient of α_1 has disappeared in the state of Eq.(10).

The next step is to prepare another single photon in single-photon source S_3 in the spatial mode b_3 of the form

$$
|\Phi\rangle_3 = \frac{\alpha_2}{\sqrt{\alpha_2^2+\alpha_3^2}}|V\rangle + \frac{\alpha_3}{\sqrt{\alpha_2^2+\alpha_3^2}}|H\rangle. \qquad (12)
$$

Following the same principle described above, Bob3 lets the photon of number 3 in $|\Psi^+\rangle_N^1$ in the spatial mode a_3 combined with the single photon $|\Phi\rangle_3$ in the spatial mode b_3 pass through his PPC gate. Then the whole system evolves to

$$
\begin{aligned}
|\Psi^+\rangle_N^1 \otimes |\Phi\rangle_3 &\rightarrow \frac{\alpha_2\alpha_3}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}\sqrt{\alpha_2^2+\alpha_3^2}}|V\rangle_1|H\rangle_2|H\rangle_3|H\rangle|\widetilde{H}\rangle^{N-3}\\
&+ \frac{\alpha_2\alpha_3}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}\sqrt{\alpha_2^2+\alpha_3^2}}|H\rangle_1|V\rangle_2|H\rangle_3|H\rangle|\widetilde{H}\rangle^{N-3}\\
&+ \frac{\alpha_2\alpha_3}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}\sqrt{\alpha_2^2+\alpha_3^2}}|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle|\widetilde{H}\rangle^{N-3}\\
&+ \cdots\\
&+ \frac{\alpha_3\alpha_N}{\sqrt{2\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2}\sqrt{\alpha_2^2+\alpha_3^2}}|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle|\widetilde{H}\rangle^{N-3}|V\rangle_N, \qquad (13)
\end{aligned}
$$

If he picks up the even parity state, then Bob3 measures the photon in the spatial mode d_3 in the basis $|\pm\rangle$. They will get

$$\begin{aligned}
|\Psi^{\pm}\rangle_N^3 &= \frac{\alpha_2}{\sqrt{3\alpha_2^2+\alpha_4^2+\cdots\alpha_N^2}}|V\rangle_1|H\rangle_2|H\rangle_3|\widetilde{H}\rangle^{N-3} \\
&+\frac{\alpha_2}{\sqrt{3\alpha_2^2+\alpha_4^2+\cdots\alpha_N^2}}|H\rangle_1|V\rangle_2|H\rangle_3|H\rangle|\widetilde{H}\rangle^{N-3} \\
&\pm\frac{\alpha_2}{\sqrt{3\alpha_2^2+\alpha_4^2+\cdots\alpha_N^2}}|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3} \\
&+\frac{\alpha_4}{\sqrt{3\alpha_2^2+\alpha_4^2+\cdots\alpha_N^2}}|H\rangle_1|H\rangle_2|H\rangle_3|V\rangle_4|\widetilde{H}\rangle^{N-4} \\
&+\cdots \\
&+\frac{\alpha_N}{\sqrt{3\alpha_2^2+\alpha_4^2+\cdots\alpha_N^2}}|H\rangle_1|H\rangle_2|H\rangle_3|\widetilde{H}\rangle^{N-4}|V\rangle_N.
\end{aligned} \tag{14}$$

The total success probability is

$$P^3 = \frac{3\alpha_2^2\alpha_3^2+\alpha_3^2(\alpha_4^2+\cdots+\alpha_N^2)}{(2\alpha_2^2+\alpha_3^2+\alpha_4^2+\cdots+\alpha_N^2)(\alpha_2^2+\alpha_3^2)}. \tag{15}$$

P^3 essentially contains two parts. The first one is the success probability to get $|\Psi^{\pm}\rangle_N^1$, and the second one is the success probability for Bob3 to pick up the even parity state. Interestingly, from Eq. (14), the coefficient α_3 has also disappeared. The following concentration steps are similar to the above description. That is each one performs a parity check measurement and picks up the even parity state. For instance, in the Kth step, Bob$K$ first prepares a single photon of the form

$$|\Phi\rangle_K = \frac{\alpha_2}{\sqrt{\alpha_2^2+\alpha_K^2}}|V\rangle+\frac{\alpha_K}{\sqrt{\alpha_2^2+\alpha_K^2}}|H\rangle. \tag{16}$$

After he performing the parity check measurement and picks up the even parity state, the original less-entangled W state becomes

$$\begin{aligned}
|\Psi^{\pm}\rangle_N^K &= \frac{\alpha_2}{\sqrt{K\alpha_2^2+\alpha_{K+1}^2+\cdots\alpha_N^2}}|V\rangle_1|\widetilde{H}\rangle^{N-1} \\
&+\frac{\alpha_2}{\sqrt{K\alpha_2^2+\alpha_4^2+\cdots\alpha_N^2}}|H\rangle_1|V\rangle_2|H\rangle_3|\widetilde{H}\rangle^{N-3} \\
&+\cdots \\
&\pm\frac{\alpha_2}{\sqrt{K\alpha_2^2+\alpha_{K+1}^2+\cdots\alpha_N^2}}|\widetilde{H}\rangle^{K-1}|V\rangle_K|\widetilde{H}\rangle^{N-K} \\
&+\frac{\alpha_{K+1}}{\sqrt{K\alpha_2^2+\alpha_{K+1}^2+\cdots\alpha_N^2}}|\widetilde{H}\rangle^{K}|V\rangle_{K+1}|\widetilde{H}\rangle^{N-K-1} \\
&+\cdots \\
&+\frac{\alpha_N}{\sqrt{K\alpha_2^2+\alpha_{K+1}^2+\cdots\alpha_N^2}}|\widetilde{H}\rangle^{N-1}|V\rangle_N.
\end{aligned} \tag{17}$$

The success probability can be written as

$$P^K = \frac{K\alpha_2^2\alpha_K^2+\alpha_K^2(\alpha_{K+1}^2+\alpha_{K+2}^2\cdots+\alpha_N^2)}{((K-1)\alpha_2^2+\alpha_K^2+\alpha_{K+1}^2+\cdots+\alpha_N^2)(\alpha_2^2+\alpha_K^2)}. \tag{18}$$

If $K=N$, then they will get the maximally entangled W state, with the probability of

$$P^N = \frac{N\alpha_2^2\alpha_N^2}{((N-1)\alpha_2^2+\alpha_N^2)(\alpha_2^2+\alpha_N^2)}. \tag{19}$$

Therefore, the total success probability to get the maximally entangled W state from Eq. (5) is

$$P_T = P^1P^3\cdots P^N = \frac{N\alpha_1^2\alpha_2^2\alpha_3^2\cdots\alpha_N^2}{(\alpha_2^2+\alpha_1^2)(\alpha_2^2+\alpha_3^2)\cdots(\alpha_2^2+\alpha_N^2)}. \tag{20}$$

Interestingly, if $N=2$, it is the concentration of two-particle Bell state with $P_T=2\alpha_1^2\alpha_2^2$. It is equal to the success probability in Refs.[33, 35, 36, 38].

By far, we have fully explained our ECP with PPC gate. During the whole process, we require $N-1$ single photons to achieve this task with the success probability of P_T. Except Bob2, each parties needs to perform a parity check. If the parity check measurement result is even, it is successful and he asks others to retain their photons. From Sec. II, the PPC gate essentially is based on linear optics and we should resort the post selection principle. That is to say, the detection will destroy their photons. This disadvantage will greatly limits its practical application, because it has to require all of the parties to perform the parity check simultaneously. On the other hand, the total success probability is extremely low. Because they should ensure all $N-1$ parity checks be successful. If any of parity check in Bob$K$ is fail, then the whole ECP is fail. It is quite different from the ECP of N-particle GHZ state[33, 35, 36], due to the same entanglement structure with Bell state. The ECP of Bell state is suitable to the N-particle GHZ state with the same success probability $2\alpha_1^2\alpha_2^2$ with linear optics[33]. That is to say, the success probability does not change with the particle number N. However, in this ECP, we find that the P_T changes when N changes.

B. N-particle W state concentration with CPC gate

From above description, we show that the PPC gate can be used to achieve this concentration task. However, it is not an economical one and the success probability is extremely low. The reason is that we only pick up the even parity state and discard the odd one. In this section, we will adopt the PPC gate to redescribe this ECP. The basic principle of our ECP is shown in Fig. 4. We use the CPC gates to substitute the PPC gates. In the first step, the initial state $|\Psi\rangle_N$ and $|\Phi\rangle_1$ combined with the coherent state $|\alpha\rangle$ evolve as

$$\begin{aligned}
&|\Psi\rangle_{N+1}\otimes|\alpha\rangle = |\Psi\rangle_N\otimes|\Phi\rangle_1 = (\alpha_1|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2} \\
&+\ \alpha_2|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}+\cdots+\alpha_N|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N) \\
&\otimes\ (\frac{\alpha_1}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle+\frac{\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle)\otimes|\alpha\rangle
\end{aligned}$$

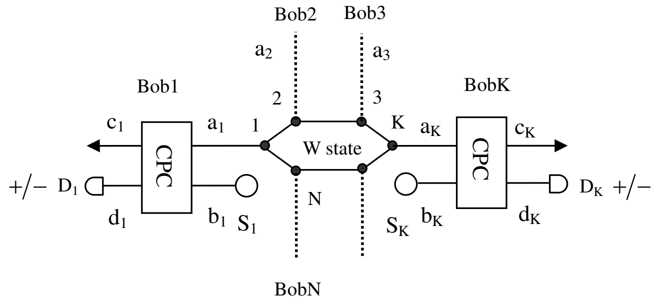

FIG. 4: A schematic drawing of our ECP with CPC gate. Compared with Fig. 3, we use the CPC gate shown in Fig. 2 to substitute the PPC gate. By using the CPC gate, the odd parity state can also be reused to improve the success probability and the concentrated state can also be retained.

$$\begin{aligned}
&\rightarrow \frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\otimes|\alpha e^{-i2\theta}\rangle\\
&+ \frac{\alpha_2^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\otimes|\alpha e^{i2\theta}\rangle\\
&+ \frac{\alpha_1\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\otimes|\alpha\rangle\\
&+ \frac{\alpha_1\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\otimes|\alpha\rangle\\
&+ \cdots\\
&+ \frac{\alpha_1\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N\otimes|\alpha\rangle\\
&+ \frac{\alpha_1\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\otimes|\alpha\rangle\\
&+ \frac{\alpha_2\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\otimes|\alpha e^{-i2\theta}\rangle\\
&+ \cdots\\
&+ \frac{\alpha_2\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N\otimes|\alpha e^{-i2\theta}\rangle. \quad (21)
\end{aligned}$$

Obviously, if the coherent state $|\alpha\rangle$ picks up no phase shift, the original state will collapse to the even state similar to $|\Psi\rangle'_{N+1}$ in Eq. (8). It can also be rewritten as $|\Psi\rangle''_{N+1}$ with the probability of P^1. In this way, they can also obtain the same state $|\Psi^{\pm}\rangle_N^1$ in Eq. (10) and can be used to start the next concentration step on the number 3 photon performed by Bob3. On the other hand, there is the probability of $1-P^1$ that the original state will collapse to the odd state, if the phase shift of coherent state is 2θ. Therefore, it can be written as

$$\begin{aligned}
|\Psi_{\perp}\rangle'_{N+1} &= \frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\rangle\\
&+ \cdots\\
&+ \frac{\alpha_2\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N.
\end{aligned} \quad (22)$$

After measuring the photon in d_1 mode in the basis $|\pm\rangle$, above state becomes

$$\begin{aligned}
|\Psi_{\perp}^{\pm}\rangle'_N &= \pm\frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2^2}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2\alpha_3}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\rangle\\
&+ \cdots\\
&+ \frac{\alpha_2\alpha_N}{\sqrt{\alpha_1^2+\alpha_2^2}}|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N.
\end{aligned} \quad (23)$$

If the measurement result is $|+\rangle$, they will get $|\Psi_{\perp}^{+}\rangle'_N$, otherwise, they will get $|\Psi_{\perp}^{-}\rangle'_N$. Above equation can be rewritten as

$$\begin{aligned}
|\Psi_{\perp}^{\pm}\rangle''_N &= \pm\frac{\alpha_1^2}{T}|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2^2}{T}|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2\alpha_3}{T}|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\rangle\\
&+ \cdots\\
&+ \frac{\alpha_2\alpha_N}{T}|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N. \quad (24)
\end{aligned}$$

$T=\sqrt{\alpha_1^4+\alpha_2^2(\alpha_2^2+\alpha_3^2+\cdots+\alpha_N^2)}$. Interestingly, the state of Eq. (24) essentially is a lesser-entangled W state. It can be reconcentrated with another single photon on the number 1 photon. The another single photon is written as

$$|\Phi\rangle'_1=\frac{\alpha_1^2}{\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle+\frac{\alpha_2^2}{\sqrt{\alpha_1^4+\alpha_2^4}}|V\rangle. \quad (25)$$

So Bob1 can restart this ECP with the help of a second single photon $|\Phi\rangle'_1$. The state $|\Psi_{\perp}^{+}\rangle''_N$ and $|\Phi\rangle'_1$ combined with the coherent state $|\alpha\rangle$ evolves as

$$\begin{aligned}
&|\Psi_{\perp}^{+}\rangle''_N\otimes|\Phi\rangle'_1\otimes|\alpha\rangle = \frac{\alpha_1^2}{T}|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2^2}{T}|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}\\
&+ \frac{\alpha_2\alpha_3}{T}|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\rangle\\
&+ \cdots\\
&+ \frac{\alpha_2\alpha_N}{T}|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N\\
&\otimes\ (\frac{\alpha_1^2}{\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle+\frac{\alpha_2^2}{\sqrt{\alpha_1^4+\alpha_2^4}}|V\rangle)\otimes|\alpha\rangle
\end{aligned}$$

$$
\begin{aligned}
\rightarrow\ & \frac{\alpha_1^4}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}|\alpha e^{-i2\theta}\rangle \\
+\ & \frac{\alpha_1^2\alpha_2^2}{T\sqrt{\alpha_1^4+\alpha_2^4}}|V\rangle|V\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-2}|\alpha\rangle \\
+\ & \frac{\alpha_1^2\alpha_2^2}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}|\alpha\rangle \\
+\ & \frac{\alpha_1^4}{T\sqrt{\alpha_1^4+\alpha_2^4}}|V\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2}|\alpha e^{i2\theta}\rangle \\
+\ & \frac{\alpha_1^2\alpha_2\alpha_3}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\rangle|\alpha\rangle \\
+\ & \cdots \\
+\ & \frac{\alpha_1^2\alpha_2\alpha_N}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N|\alpha\rangle \\
+\ & \frac{\alpha_2^3\alpha_3\alpha_N}{T\sqrt{\alpha_1^4+\alpha_2^4}}|V\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}|\alpha e^{i2\theta}\rangle \\
+\ & \cdots \\
+\ & \frac{\alpha_2^3\alpha_N\alpha_N}{T\sqrt{\alpha_1^4+\alpha_2^4}}|V\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N|\alpha e^{i2\theta}\rangle
\end{aligned}
\tag{26}
$$

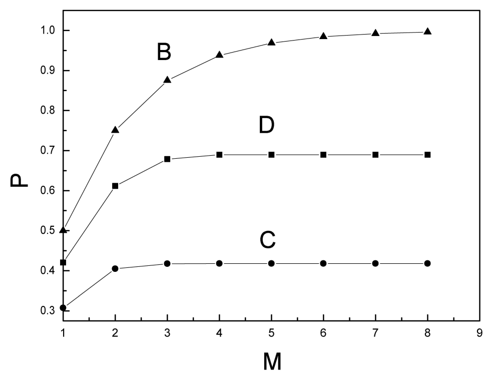

FIG. 5: The success probability of concentration of each photon in each step is altered with the iteration number M. Here we choose the five-photon less entangled W state with $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, $\alpha_4 = 0.3$, $\alpha_5 = 0.4$. Curve B: the success probability of concentration the number 1 and 3 photons according to $\alpha_1 = \alpha_3 = 0.5$. Curve C: the success probability of concentration the number 4 photon according to $\alpha_4 = 0.3$. Curve D: the success probability of concentration the number 4 photon according to $\alpha_5 = 0.4$.

Obviously, if Bob1 picks up no phase shift, above equation collapses to

$$
\begin{aligned}
|\Psi_\perp\rangle'_{N+1} =\ & \frac{\alpha_1^2\alpha_2^2}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2} \\
+\ & \frac{\alpha_1^2\alpha_2^2}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|V\rangle_2|\widetilde{H}\rangle^{N-2} \\
+\ & \frac{\alpha_1^2\alpha_2\alpha_3}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|H\rangle_2|V\rangle_3|\widetilde{H}\rangle^{N-3}\rangle|\alpha\rangle \\
+\ & \cdots \\
+\ & \frac{\alpha_1^2\alpha_2\alpha_N}{T\sqrt{\alpha_1^4+\alpha_2^4}}|H\rangle|H\rangle_1|H\rangle_2|\widetilde{H}\rangle^{N-3}|V\rangle_N.
\end{aligned}
\tag{27}
$$

Interestingly, above state essentially is the state $|\Psi\rangle''_{N+1}$

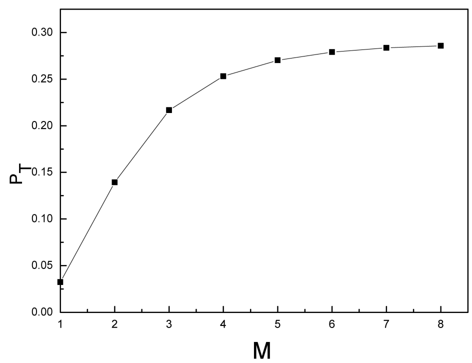

FIG. 6: The total success probability of our ECP altered with iteration number M with CPC gate. We also let $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, $\alpha_4 = 0.3$, $\alpha_5 = 0.4$.

in Eq.(9), if it is normalized. Then it can be used to concentrate the number 3 photon with the same CPC gate like above. The success probability is

$$
P_2^1 = \frac{2\alpha_1^4\alpha_2^4 + \alpha_1^4\alpha_2^2(\alpha_3^2+\alpha_4^2+\cdots+\alpha_N^2)}{(\alpha_1^2+\alpha_2^2)(\alpha_1^4+\alpha_2^4)}. \tag{28}
$$

Following the same principle, Bob1 can repeat this ECP for M times and they can get the success probability in each step as

$$
\begin{aligned}
P_3^1 &= \frac{2\alpha_1^8\alpha_2^8 + \alpha_1^8\alpha_2^6(\alpha_3^2+\alpha_4^2+\cdots+\alpha_N^2)}{(\alpha_1^2+\alpha_2^2)(\alpha_1^4+\alpha_2^4)(\alpha_1^8+\alpha_2^8)}, \\
&\cdots \\
P_M^1 &= \frac{2\alpha_1^{2^M}\alpha_2^{2^M} + \alpha_1^{2^M}\alpha_2^{2^M-2}(\alpha_3^2+\alpha_4^2+\cdots+\alpha_N^2)}{(\alpha_1^2+\alpha_2^2)(\alpha_1^{2^2}+\alpha_2^{2^2})\cdots(\alpha_1^{2^M}+\alpha_2^{2^M})}.
\end{aligned}
\tag{29}
$$

Here the superscription 1 means that concentration on the number 1 photon. The subscription M means that the ECP is performed M times.

After performing the concentration ECP on the number 1 photon, they will have a total success probability with $P_{total}^1 = P_1^1 + P_2^1 + \cdots + P_M^1 = \sum_{M=1}^{\infty} P_M^1$ to obtain $|\Psi^\pm\rangle_N^1$, which can be used to performing the concentration scheme on the number 3 photon. So far, we have explained our ECP performed on the number 1 photon with CPC gate. Different from the scheme described with PPC gate, it can be repeated to get a high success probability.

Following the same principle, they can also use this way to concentrating each photons. If they perform this

ECP on the Kth ($K \neq 2$) photon with M times, they can get the success probability P_M^K

$$P_M^K = \frac{K\alpha_2^{2^M}\alpha_K^{2^M} + \alpha_K^{2^M}\alpha_2^{2^M-2}(\alpha_{K+1}^2 + \alpha_{K+2}^2 + \cdots + \alpha_N^2)}{[(K-1)\alpha_2^2 + \alpha_K^2 + \alpha_{K+1}^2 + \cdots \alpha_N^2][(\alpha_2^2 + \alpha_K^2)(\alpha_2^{2^2} + \alpha_K^{2^2})\cdots(\alpha_2^{2^M} + \alpha_K^{2^M})]}. \tag{30}$$

If $K = N$, they can get

$$P_M^N = \frac{N\alpha_2^{2^M}\alpha_N^{2^M}}{[(N-1)\alpha_2^2 + \alpha_N^2][(\alpha_2^2 + \alpha_N^2)(\alpha_2^{2^2} + \alpha_N^{2^2})\cdots(\alpha_2^{2^M} + \alpha_N^{2^M})]}. \tag{31}$$

Therefore, if we use the CPC gate to perform the EPC, each parties can repeat this ECP to increase the success probability. Suppose each one all perform this ECP for M times, the success probability of get a maximally entangled W state from the initial state in Eq. (5) can be described as

$$\begin{aligned} P &= P_{total}^1 P_{total}^3 P_{total}^4 \cdots P_{total}^N \\ &= (P_1^1 + P_2^1 + \cdots + P_M^1)(P_1^3 + P_2^3 \\ &+ \cdots + P_M^3)\cdots(P_1^N + P_2^N + \cdots + P_M^N) \\ &= \prod_{K=1, K\neq 2}^{N}(\sum_{M=1}^{\infty} P_M^K). \end{aligned} \tag{32}$$

Compared with the ECP with PPC gate, the success probability in Eq. (20) is the case of $M = 1$ in Eq. (32).

In Fig. 5, we show that the success probability of concentration of each photon altered with the iteration number M. We take the five-photon less-entangled W state as an example. We let $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, $\alpha_4 = 0.3$ and $\alpha_5 = 0.4$. Interestingly, if $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, the success probability of concentration number 1 photon P_M^1 is equal to P_M^3, shown in Curve B. We calculated the total success probability of our protocol with CPC gate shown in Fig.6. It is shown that, if we use the PPC gate, the success probability is the case of $M = 1$, that is 0.03228. But if we use the CPC gate and iterate it for eight times, the success probability can be increased to 0.28575. It is about nine times greater than the success probability of using PPC gate.

IV. DISCUSSION AND SUMMARY

So far, we have fully described our ECP for N-particle less-entangled W state. We explain this ECP with two different methods. The first one is to use the PPC gates and the second one is to use the CPC gates. In our ECP, after successfully performing this parity check, all coefficients in the initial state are equal to α_2. In fact, this is not the unique way to achieve this task. We can also choose α_1 and make all coefficients be equal to α_1 after performing this ECP. Choosing different coefficients do not change the basic principle of this ECP, but it will change the total success probability. In detail, we take four-particle less-entangled W state and five-particle less-entangled W state for example. Fig. 7 shows the success probability altering with the iteration number M for case of four-particle. In Fig. 7, the less-entangled W states corresponding to different curves essentially have the same entanglement. Because they can change to each other with local operations. However, it is shown that the same initial entanglement have the different success

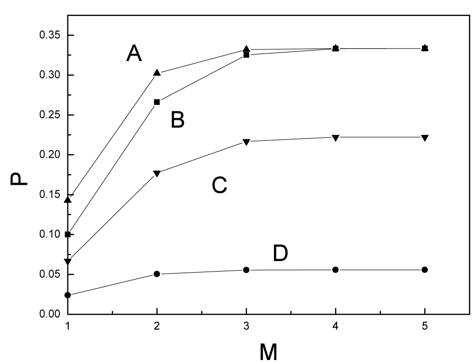

FIG. 7: The total success probability of our ECP for four-partite W state altered with iteration number M with CPC gate. Curve A: $\alpha_1 = \frac{1}{\sqrt{6}}$, $\alpha_2 = \frac{1}{\sqrt{12}}$, $\alpha_3 = \frac{1}{\sqrt{2}}$, $\alpha_4 = \frac{1}{2}$. Curve B: $\alpha_1 = \frac{1}{2}$, $\alpha_2 = \frac{1}{\sqrt{6}}$, $\alpha_3 = \frac{1}{\sqrt{2}}$, $\alpha_4 = \frac{1}{2}$. Curve C: $\alpha_1 = \frac{1}{\sqrt{2}}$, $\alpha_2 = \frac{1}{2}$, $\alpha_3 = \frac{1}{\sqrt{6}}$, $\alpha_4 = \frac{1}{\sqrt{12}}$. Curve D: $\alpha_1 = \frac{1}{\sqrt{12}}$, $\alpha_2 = \frac{1}{\sqrt{2}}$, $\alpha_3 = \frac{1}{2}$, $\alpha_1 = \frac{1}{\sqrt{6}}$.

probabilities if we choose different α_2. In Fig. 8, we also calculate the similar case of five-particle less-entangled W state. One can see that choosing different α_2 leads different total success probability. That is α_2 smaller, the total success probability is greater. We can explain this result

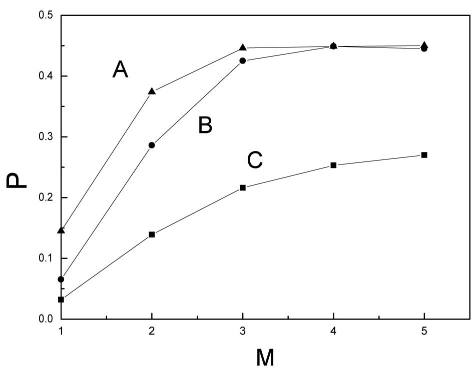

FIG. 8: The total success probability of our ECP for five-particle W state altered with iteration number M with CPC gate. Curve A: $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = \alpha_4 = \alpha_5 = 0.5$. Curve B: $\alpha_1 = 0.5$, $\alpha_2 = 0.4$, $\alpha_3 = 0.3$, $\alpha_4 = \alpha_5 = 0.5$. Curve C: $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, $\alpha_4 = 0.3$, $\alpha_5 = 0.4$.

from Eq. (20) and Eq. (31). In Eq. (20), if $\alpha_1, \alpha_2, \cdots \alpha_N$ are given, the numerator is a constant. But the value of denominator is decided by α_2. Therefore, choosing the smallest value of α_2 will get the highest success probability. This result provide us an useful way to performing this ECP. If α_2 is not the smallest one, then we can rotate the polarization of each photon with half-wave plate until to obtain the smallest α_2 . Certainly, we should point out that the ECP with PPC gate and with CPC gate are quite different form each other in the practical manipulation. Because the PPC gate is equipped with the linear optical elements and we should resort the pose selection principle to achieve this task. That is, after successfully performed this ECP, the maximally entangled W state is destroyed by the sophisticated single photon detectors. This condition greatly limit its practical application. In addition, in Sec. III A, we explain it by dividing the whole ECP into $N-1$ steps. In each step, one of the parties prepares one single photon and makes a parity check measurement. Practically, each parties except Bob2 should perform the parity check measurement simultaneously due to the post selection principle. If all parity check measurements are even parities, then by classical communication, they ask each to retain their photons, and it is a successful case. On the other hand, if we adopt CPC gate to perform this ECP, each parties can operate his photons independently. That is, each one can repeat to perform concentration until it is successful. The most fundamental reason is that QND is only to check the phase shift of the coherent state and it does not destroy the photon after measurement. In our ECP, after performing the parity check measurement using CPC gate, the next operation is decided by the measurement result. If it is even parity, it is successful, otherwise, each one can restart to concentrate his photon with another single photon. This strategy makes the total concentration efficiency be greatly improved.

Finally, let us discuss the key element of our ECP, that is the cross-Kerr nonlinearity. In Ref. [35] and [37], they also adopt the cross-Kerr nonlinearity to construct the parity check gate to achieve the concentration tasks. Unfortunately, in order to increase the efficiency of the protocol, they should resort the coherent state to obtain π phase shift. Although there are several strategies to increase the phase shift, such as increasing the strength of the coherent state, controlling the coupling time of the coherent state and the Kerr media, and choosing the suitable Kerr media, to get giant phase shift is still difficult in current technology [57, 58]. Meanwhile, cross-Kerr nonlinearity is also a controversial topic. The focus of the argument is still that one cannot get giant phase shift on the single-photon level. This conclusion is agree with the results of Shapiro, Razavi, and Gea-Banacloche[59–61]. On the other hand, Hofmann pointed out that with a single two-level atom in a one-sided cavity, a large phase-shift of π can be achieved[62]. Current research showed that it is possible to amplify a cross-Kerr-phase-shift to an observable value by using weak measurements, which is much larger than the intrinsic magnitude of the single-photon-level nonlinearity[63]. Zhu and Huang also discussed the possibility of obtain the giant cross-Kerr nonlinearities using a double-quantum-well structure with a four-lever, double-type configuration[64]. Fortunately, we do not require the coherent state to get π phase shift. It is an improvement of Refs. [35–37]. This kind of parity check gate is first used to perform the entanglement purification in Ref. [24]. Then Guo *et al.* developed this idea, and used it to perform the Bell-state analyzer, prepare the cluster-state and so on [56]. As discussed by Guo *et al.*, compared with the previous parity check gate[35–37], it has several advantages: first, it is an effective simplification by removing two PBSs and several mirrors; second, it has a lower error rate. Third, it does not require the π phase shift which is more suitable in current experimental conditions.

In summary, we have present an universal way to concentrate an N-particle less-entangled W state into a maximally entangled W state with both PPC gate and CPC gate. In the former, we require the linear optical elements and post selection principle. In the later, we use cross-Kerr nonlinearity to construct the QND. Different from other concentration protocols, we only need single photon as an auxiliary to achieve the task. Then this ECP does not largely consume the less-entangled photon systems. Especially, with the help of QND, each parties can operated independently and this ECP can be repeated to get a higher success probability. These advantages may make this ECP more useful in practical application in current quantum information processing.

ACKNOWLEDGEMENTS

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